Parity doublets and chiral symmetry restoration in baryon spectrum

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Abstract

It is argued that an appearance of the near parity doublets in the upper part of the light baryon spectrum is an evidence for the chiral symmetry restoration in the regime where a typical momentum of quarks is around the chiral symmetry restoration scale. At high enough baryon excitation energy the nontrivial gap solution, which signals the chiral symmetry breaking regime, disappears and the chiral symmetry should be restored. Thus one observes a phase transition in the upper part of the light baryon spectrum. The average kinetic energy of the constituent quarks in this region is just around the critical one $3T_c$.

The still poorly mapped upper part of the light baryon spectrum [1] exhibits remarkable parity doublet patterns. To these belong $N(2220), \frac{9}{2}^+ - N(2250), \frac{9}{2}^-$, $N(1990), \frac{7}{2}^+ - N(2190), \frac{7}{2}^-$, $N(2000), \frac{5}{2}^+ - N(2200), \frac{5}{2}^-$, $N(1900), \frac{3}{2}^+ - N(2080), \frac{3}{2}^-$, $\Delta(2300), \frac{9}{2}^+ - \Delta(2400), \frac{9}{2}^-$, $\Delta(1950), \frac{7}{2}^+ - \Delta(2200), \frac{7}{2}^-$, $\Delta(1905), \frac{5}{2}^+ - \Delta(1930), \frac{5}{2}^-$, $\Delta(1920), \frac{3}{2}^+ - \Delta(1940), \frac{3}{2}^-$, $\Delta(1910), \frac{1}{2}^+ - \Delta(1900), \frac{1}{2}^-$. The splittings within the parity partners are typically within the 5% of the baryon mass. This value should be given a large uncertainty range because of the experimental uncertainties for the baryon masses of the order 100 MeV. Only a couple of states in this part of the spectrum do not have their parity partners so far observed. The low energy part of the spectrum, on the other hand, does not show this property of the parity doubling. The increasing amount of the near parity doublets in the high energy sector was a motivation to speculate that the baryon spectrum exhibits a smooth transition from the Nambu-Goldstone mode of chiral symmetry in the low-energy part to the Wigner-Weyl mode in the upper part [2]. In this note I shall give a further impetus to this idea and show that this phenomenological observation has in fact a simple and transparent microscopical foundation.

The almost perfect $SU(2)_L \times SU(2)_R$ global chiral symmetry of the QCD Lagrangian in the u, d sector is equivalent to the independent vector and axial rotations in the isospin

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space. The axial transformation mixes states with different spatial parities. Hence, if this symmetry of the QCD Lagrangian were intact in the vacuum, one would observe parity degeneracy of all hadron states with otherwise the same quantum numbers. This is however not so and it was a reason for suggestion in the early days of QCD that the chiral symmetry of the QCD Lagrangian is broken down to the vectorial subgroup $SU(2)_V$ by the QCD vacuum, which reflects a conservation of the vector current (baryon number). That this is so is directly evidenced by the nonzero value of the quark condensate

$$< \bar{\psi}\psi > \simeq -(240 - 250 MeV)^3, \tag{1}$$

which represents the order parameter associated with the chiral symmetry breaking. The nonzero value of the quark condensate directly shows that the vacuum state is not chiral-invariant.

Physically the nonzero value of the quark condensate implies that the energy of the state which contains an admixture of "particle-hole" excitations (real vacuum of QCD) is below the energy of the vacuum for a free Dirac field, in which case all the negative-energy levels are filled in and all the positive-energy levels are free. This can happen only due to some nonperturbative gluonic interactions between quarks which pairs the left quarks and the right antiquarks (and vice versa) in the vacuum. In perturbation theory to any order the structure of the trivial Dirac vacuum persists. Such a situation is typical in many-fermion systems (compare, e.g., with the theory of superconductivity) and implies that there must appear quasiparticles with dynamical masses. While there are indications that the instanton fluctuations of the gluonic field could be important for the chiral symmetry breaking and thus for the creation of quasiparticles [3], the issue on the true dynamical mechanism of the chiral symmetry breaking in QCD is still unclear. For example, the nonperturbative resummation of gluonic exchanges by solving the Schwinger-Dyson equation [4] is known to also lead to chiral symmetry breaking if the strong coupling constant is big enough (however, in the latter case the $U(1)_A$ problem persists), or it can be generated by monopole condensation, which is a fashionable scenario for the string-like confining force in QCD.

Formally the quark condensate represents the closed loop in momentum space, with the fermion line beginning and ending at the same space-time point:

$$< \bar{\psi}\psi > = -i \; tr S_F(0), \tag{2}$$

where $S_F(x-y)$ is Dirac Green function:

$$S_F(x-y) = -i < T[\psi(x)\bar{\psi}(y)] > = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \frac{\not{p} + m}{p^2 - m^2 + i\epsilon}. \tag{3}$$

The trace over slash term gives identically zero. Hence the nonzero value of the quark condensate implies that the massless quark field ($m = 0$) acquires a non-zero dynamical mass, $M(p)$, which should be in general a momentum-dependent quantity

$$< \bar{\psi}\psi >= -4N_c i \int \frac{d^4p}{(2\pi)^4} \frac{M(p)}{p^2 - M^2(p) + i\epsilon}. \tag{4}$$

The dynamical mass $M(p)$ also represents the order parameter. This mass should vanish at high momentum, in which case the quarks are not influenced by the QCD medium and perturbative QCD is applicable. But at small momenta, below the chiral symmetry breaking scale, Λ_χ, the QCD nonperturbative phenomena become crucial and give rise to the nonzero dynamical mass as well as to the condensate. This dynamical mass at small momenta can be evidently linked to the constituent mass of quarks introduced in the context of naive quark model [5]. It is this type of behavior of dynamical mass which is observed on the lattice [6].

As soon as the chiral symmetry is dynamically broken at low momenta, then necessarily appear Goldstone bosons which couple to constituent quarks [7]. This property is most transparently illustrated by the sigma-model [8] and Nambu and Jona-Lasinio (NJL) model [9]. The latter one suggests an insight into chiral symmetry breaking, constituent mass generation, Nambu-Goldstone bosons as collective quark-antiquark modes. This model is known to be very successfull in the low-lying meson spectrum, for reviews see [10]. It approximates a smooth drop of the dynamical mass $M(p)$ by a step function, see Fig. 1 (because of a local character of the effective 4-fermion interaction in this model). Thus at momenta below chiral symmetry breaking scale Λ_χ, which corresponds to the ultraviolet cut-off within the Nambu and Jona-Lasinio model, the adequate effective degrees of freedom are the constituent quarks and Goldstone bosons coupled to each other.

It is instructive to review some of the basic properties of the NJL model, which is nothing else but Bardeen-Cooper-Scrieffer theory of superconductivity, extended to explain chiral symmetry breaking and dynamical mass generation.

Any scalar gluonic interaction between current quarks, which is responsible for the chiral symmetry breaking in QCD, in the *local* approximation is given by the 4-fermion operator $(\bar{\psi}\psi)^2$. Because of the underlying chiral invariance in QCD this interaction should be necessarily accompanied by the interaction $(\bar{\psi}i\gamma_5\vec{\tau}\psi)^2$ with the same strength. Thus any generic Hamiltonian density in the local approximation is given by (contains as a part) the NJL interaction model (for simplicity we restrict discussion to the u,d flavour sector and to the chiral limit):

$$H = -G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]. \tag{5}$$

If the strength of the interaction G exceeds some critical level, then the nonlinear *gap* equation

$$M = -2G < \bar{\psi}\psi > , \tag{6}$$

$$< \bar{\psi}\psi >= -\frac{N_c}{\pi^2}\int_0^{\Lambda_\chi} d|\mathbf{p}|\mathbf{p}^2 \frac{M}{\sqrt{(\mathbf{p}^2 + M^2)}} \tag{7}$$

admits a nontrivial solution $< \bar{\psi}\psi > \neq 0$, which means that the initial vacuum becomes rearranged and instead of the Wigner-Weyl mode of chiral symmetry one obtains the Nambu-Goldstone one. Thus the appearance of the quark condensate is equivalent to the appearance of the gap in the spectrum of elementary excitations in QCD (i.e. of the constituent mass M). Hence the constituent quark is a quasiparticle in the Bogoliubov sense, i.e. is a coherent superposition of the bare particle-hole excitations. In terms of the noninterating constituent quarks the vacuum is again trivial, but contains a gap 2M. The treatment of the scalar interaction between bare quarks in the vacuum in the Hartree-Fock (mean field) approximation, from which the gap equation (6)-(7) is obtained, is equivalent to the vacuum of *noninteracting* constituent quarks. In the vacuum state the second term of (5) does not contribute to the constituent quark self-energy in the mean field approximation. Contributions beyond the mean field approximation (i.e. constituent quark self energy due to pion and sigma loops, which are higher order effects in the $1/N_c$ expansion, see, e.g., [11]) do not violate significantly the qualitative picture of the vacuum in the mean field approximation.

In the systems that contain valence quarks on the top of the vacuum (hadrons) the situation is qualitatively different. In this case the valence quarks interact not only with the vacuum condensates (which provides their constituent mass), but also to each other. The latter interaction strongly depends on the quantum numbers of hadrons. In the pseudoscalar-isovector quark-antiquark system the second term of (5), when iterated, exactly compensates the $2M$ energy, supplied by the first term in the vacuum, and consequently there appear massless pions as Nambu-Goldstone bosons. This means that while for all other hadrons the chiral symmetry breaking implies a presence of the gap in the excitation energy ($3M$ for baryons and $2M$ for mesons), this is not the case for pions which represent a gapless excitation over QCD vacuum. Iteration of the first term in the scalar-isoscalar quark-antiquark system produces a weakly bound σ-meson with the mass $2M$. In the quark-quark systems, i.e. in baryons, iteration of the first and second terms in the qq t-channel leads to the π, σ exchange interactions between valence constituent quarks, which reprtesents effects of the strong vacuum polarization [12]. The pion-like exchange interactions between valence quarks in baryons provide the $N - \Delta$ splitting and other low-lying baryon excitations [2,13]. So all interactions between valence quarks (i.e. beyond the mean field approximation) can be reffered to as "residual", though they are very strong. In baryons to these belong π, σ exchanges and effective confining interaction between valence quarks. Which is the role of these residual interactions for the chiral symmetry restoration (breaking) phase transition?

Consider first the chiral symmetry restoration in the vacuum at high temperatures. At zero temperature the vacuum condensate is given by (7). At finite temperature it is affected and given by

$$< \bar{\psi}\psi > = -\frac{N_c}{\pi^2}\int_0^{\Lambda_\chi} d|\mathbf{p}|\mathbf{p}^2\frac{M}{E_p}[1-2n(p)], \tag{8}$$

where $E_p = \sqrt{(\mathbf{p}^2 + M^2)}$ and $n(p)$ is the Fermi-Dirac distribution function for quarks and antiquarks

$$n(p) = \frac{1}{1 + e^{\frac{E_p}{T}}}. \tag{9}$$

In the latter equation T is the temperature. At some critical temperature, T_c, the nontrivial gap solution disappears and the chiral symmetry becomes restored. Physically this is because the thermal excitations of quarks and antiquarks lead to the Pauli blocking of the levels which are necessary for the formation of the condensate. The formal (mathematical) reason is that the quark distribution function $n(p)$ is pushed out from the $p = 0$ point and becomes broad and thus affects the gap equation so that the self-consistent (gap) solution disappears. However, there could be other physical reason for the broadening of the momentum distribution. This physical reason is the strong residual interactions between valence constituent quarks in baryons.

Consider the system that contains interacting valence quarks on the top of the vacuum - baryons. The residual interaction effect can be obtained from the solution of the Schrödinger equation in the three-quark systems [13]. As outcome one gets the quark wave function (in the center of mass system) and this wave function, being squared and properly normalized should substitute $2n(p)$ in eq. (8). For the ground state 3q system (N and Δ) the one-quark wave function in momentum representation is given by the narrow bell-shaped curve and does not affect much the gap equation (8). However, the higher the radial excitation of the baryon is, the larger is the average kinetic energy (momentum) of constituent quarks in baryons , i.e. the momentum distribution becomes broader and broader. This momentum distribution is pushed out from the inner part by the nodes of the wave function. This means that at some baryon excitation energy the nontrivial gap solution of eqs. (8)-(6) should vanish and the chiral symmetry should be restored. While numerically this effect should be similar to chiral symmetry restoration at high temperature[1], physically *it is completely different as provided by the residual interactions but not by the thermal distributions.*

In the following I shall use for a qualitative estimate the fact that the chiral symmetry breaking scale is in the region

$$m_\rho < \Lambda_\chi < 4\pi f_\pi. \tag{10}$$

For constituent mass I shall take $M \sim 340$ MeV, the value which is known from 60th and which is also obtained in the recent lattice measurements [6,14]. The root mean square momentum of constituent quarks for all low-lying baryons in N and Δ spectra (with masses $\leq 1.7 - 1.8$ GeV) can be extracted from the wave functions obtained from the fit to masses in dynamical semirelativistic calculation [13]. This momentum falls into the range 500-700 MeV, which is below Λ_χ. This a-posteriori justifies a language of constituent quarks that interact via GBE and are subject to confinement in the low-lying baryons. However, this momentum is not very much below Λ_χ. Hence, baryons with mass of 2 GeV and higher where the average momentum of quarks is larger, should be in the region of the phase transition and a share of the Wigner-Weyl mode will be big enough to ensure the appearance of

[1]In both cases the critical quantity is average kinetic energy of quarks.

the near parity doublets.

This estimate is very consistent with the temperature of the phase transition , which is known from the Monte-Carlo lattice calculations, $T_c \sim 150$ MeV [15], or the most recent $T_c \sim 170 - 190$ MeV [16]. Above this critical temperature, i.e. in the chiral restored phase, the average kinetic energy of current quarks is above $3T_c$ [2]. In all low-lying light baryons with mass below 1.7 - 1.8 GeV the average kinetic energy of constituent quarks is in the region 260 - 480 MeV, which is just below the critical one. Basing on these simple arguments it is tempting to assume that one observes a phase transition in the upper part of the light baryon spectrum. If chiral and deconfinement phase transitions coincide, the conclusion should be that the highly excited baryons with masses above some critical value (where the phase transition is completed) should not exist because deconfinement phase transition should be dual to a very extensive string breaking at big separations of colour sources (colour screening). Whether this point corresponds to approximately 2.5 GeV or higher should be answered by future experiments on high baryon excitations. The phase transition can be rather broad because of the explicit chiral symmetry breaking by the nonzero value of current quark masses.

There is a couple of the well confirmed states $N(2600), \frac{11}{2}^-$ and $\Delta(2420), \frac{11}{2}^-$, in which case the parity partners are absent [1]. Thus it will be rather important to try to find them experimentally.

A few comments about the parity doubling within the potential models that attempt to describe the highly lying baryons are in order. The models that rely on confinement potential cannot explain an appearance of the systematic parity doublets. This is apparent for the harmonic confinement. The parity of the state is determined by the number N of the harmonic excitation quanta in the 3q state. The ground states (N=0) are of positive parity, all baryons from the $N = 1$ band are of negative parity, baryons from the $N = 2$ band have a positive parity irrespective of their angular momentum, etc. However, the number of states in the given band rapidly increases with N. This means that such a model cannot provide an equal amount of positive and negative parity states, which is necessary for parity doubling, irrespective of other residual interactions between quarks in such a model. Similar problem persists with the linear confinement in 3q system.

While all vacancies from the $N = 0$ and $N = 1$ bands are filled in in nature, such a model, extrapolated to the N=3 and higher bands predicts a very big amount of states, which are not observed (the so called missing resonance problem). According to the explanation suggested in the present paper the chiral restoration phase transition takes place at excitation energies typical for the $N = 3$ band (and somewhat to highest states from the $N = 2$ band). If correct, it would mean that description of baryons in this transition region

[2]This estimate is obtained with the $\sim e^{-p/T}$ distribution function for current quarks. With the Fermi-Dirac distribution function it is slightly above $3T_c$.

in terms of constituent quarks becomes inappropriate.

The model that rely on the pure color Coulomb interaction between quarks also cannot provide the systematical parity doubling. While it gives an equal amount of the positive and negative parity single quark states in the $n = 2, 4, ...$ bands (e.g. $2s-2p$, or $4s-4p$, $4d-4f$), the number of the positive parity states is always bigger in the $n = 1, 3, 5, ...$ bands.

Thus it is very important experimental task to verify whether the "missing" resonances exist or not and whether the upper part of the light baryon spectrum exhibits *systematical* parity doublet patterns.

What about meson spectra? Here the strong residual interaction due to Goldstone boson exchange is absent (it is impossible in the quark-antiquark pairs), i.e. description of excited mesons in terms of the constituent quarks interacting via chiral fields is not possible and thus the arguments above cannot be applied here. This is perhaps a reason for why there are no parity doubling patterns in meson spectra. If correct, it then means that the baryon spectrum suggests a unique opportunity to observe the chiral restoration phase transition.

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REFERENCES

[1] C. Caso et al (Particle Data Group), Europ. Phys. J. **C3** (1998) 1.

[2] L. Ya. Glozman and D.O. Riska, Physics Reports **268** (1996) 263.

[3] D. Diakonov, in:Selected Topics in Nonperturbative QCD, Proc. Enrico Fermi School, Course CXXX, A. DiGiakomo and D. Diakonov eds., Bologna (1996); hep-ph/9602375 and references therein.

[4] C.D. Roberts and A.G. Williams, Progr. Part. Nucl. Phys. **33** (1994) 477.

[5] J. Kokkedee, The Quark Model (Benjamin, New York, 1969) and original papers therein.

[6] S. Aoki et al, Phys. Rev. Lett. **82** (1999) 4392.

[7] A. Manohar and H. Georgi, Nucl. Phys. **B234** (1984) 189.

[8] M. Gell-Mann and M. Levy, Nuovo Cim. **16** (1960) 705.

[9] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345; **124** (1961) 246.

[10] U. Vogl and W. Weise, Progr. Part. Nucl. Phys., **27** (1991) 195; S. P. Klevansky, Rev. Mod. Phys., **64** (1992) 649; T. Hatsuda and T. Kunihiro, Phys. Rep. **247** (1994) 221.

[11] V. Dmitrasinovic et al, Ann. of Phys. **238** (1995) 332.

[12] L. Ya. Glozman and K. Varga, hep-ph/9901439, Phys. Rev. D - in print.

[13] L. Ya. Glozman, W. Plessas, K. Varga, R. F. Wagenbrunn, Phys. Rev. **D58**, 094030 (1998).

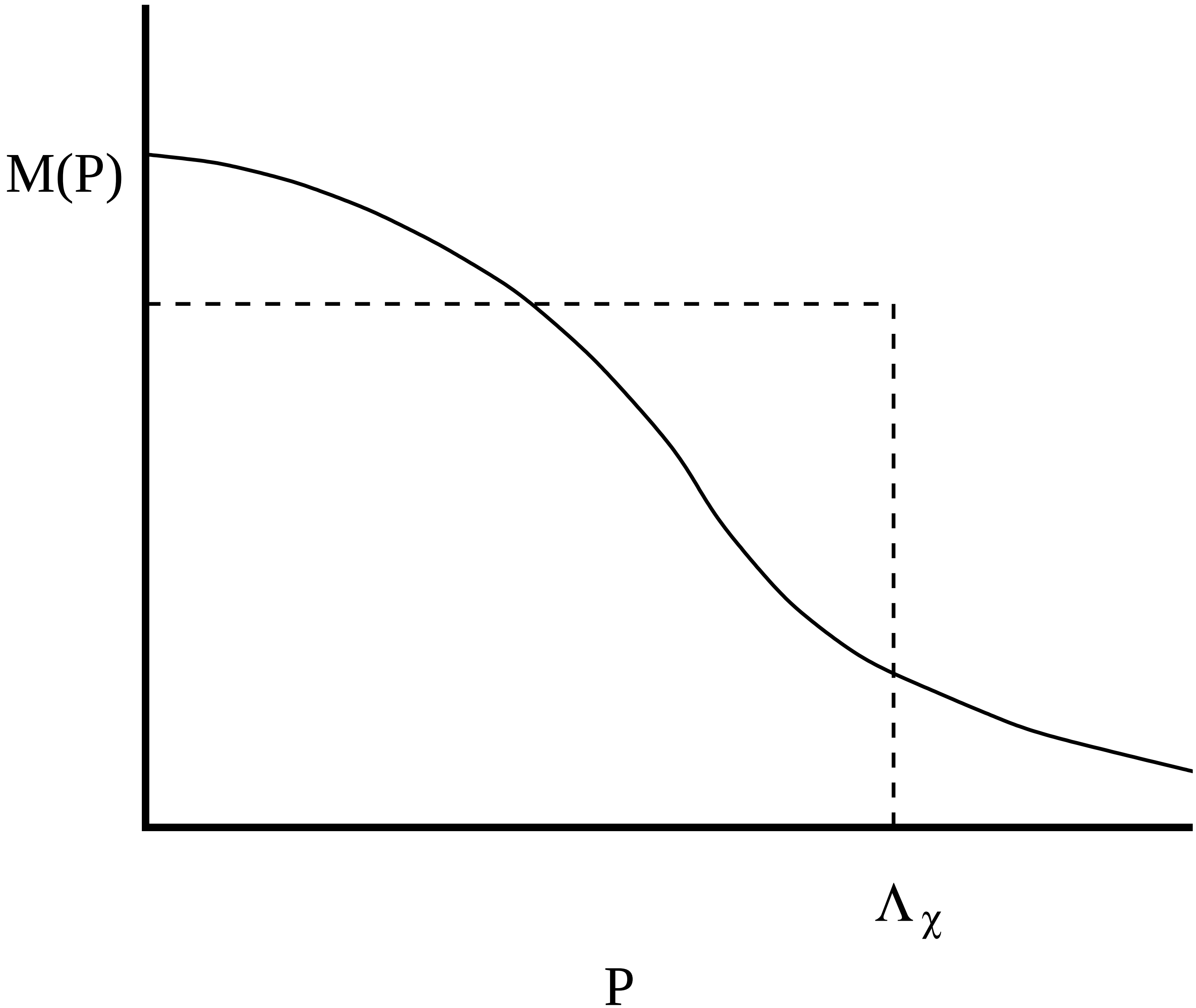
[14] M. Hess, F. Karsch, E. Laermann, I. Wetzorke, Phys. Rev. **D58** (1998) 111502.

[15] K. Kanaya, Nucl. Phys. **B (Proc. Suppl.) 47** (1996) 144; E. Laermann, Nucl. Phys. **B (Proc. Suppl.) 63A-C** (1998) 114.

[16] F. Karsch, hep-lat/9909006.

Figure captions

Fig.1 A schematic behaviour of the dynamical mass of quarks as a function of their momenta as it is expected in reality (solid line) and as it is approximated in the Nambu and Jona-Lasinio as well as in chiral quark models (dashed line).



M(P)
Λχ
P